Math 2552 Practice Problems for Midterm 2

Question 1. Find the general solution of the differential equation

$$y'' - 4y' + 4y = e^{2t} + 1.$$

Question 2.

(1) Show that $\mathbf{x}_1 = \begin{bmatrix} t \\ 1 \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} t^{\frac{1}{4}} \\ \frac{1}{4}t^{-\frac{3}{4}} \end{bmatrix}$ form a fundamental set of solutions for the homogeneous system

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{4t^2} & \frac{1}{4t} \end{bmatrix} \mathbf{x}, \ t > 0.$$

You need to check that $\mathbf{x_1}$ and $\mathbf{x}_2$ are solutions, and they are linearly independent.

(2) Find the general solution of the following non-homogeneous system of differential equations

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{4t^2} & \frac{1}{4t} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{5}{4t} \end{bmatrix}.$$

Question 3. Find the general solution of the following system of equations

$$\mathbf{x}' = \begin{bmatrix} -7 & 9 & -6 \\ -8 & 11 & -7 \\ -2 & 3 & -1 \end{bmatrix} \mathbf{x}.$$

Notice that $\lambda^3 - 3\lambda^2 + 4 = (\lambda + 1)(\lambda - 2)^2$. You need to show your work about the computation of the determinant.

Question 4. Find the fundamental matrix e^{At} where

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}.$$