Final Review Outline

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You can use a hand written crib sheet (letter size, front and back) in the final exam.

Chapter 1 and Chapter 2 - first order differential equations

Section 1.1: use differential equations for modeling, especially the tank problem.

Section 1.2 and Section 2.5: Autonomous differential equations: find equilibrium solutions (also called critical points or stationary points), draw phase lines, sketch integral curves, and determine whether a critical point is asymptotically stable, semistable or unstable. Direction fields are not on final.

Solve the following kind of first order differential equations

- 2.1: separable equations
- 2.2: first order linear differential equations
- 2.6: exact equations

Existence and uniqueness:

- Linear: Theorem 2.4.1, Theorem 3.2.1, Theorem 4.2.1, Theorem 6.2.1
- Nonlinear: Theorem 2.4.2

Section 2.7 is not on final.

Chapter 3: Systems of two first order equations

Solve the homogeneous equation $x' = Ax$: Section 3.3, 3.4 and 3.5

- Superposition principle, Wronskian and linear independence: Theorem 3.3.1, Theorem 3.3.3 and Theorem 3.3.4;
- When A has complex eigenvalues, write the solution in terms of real solutions (remember the table on Page 172).
- Draw phase portraits when A has two distinct real eigenvalues (Table 3.3.1) or complex eigenvalues (Table 3.4.1). Determine whether $(0, 0)$ is a nodal sink, a nodal source, a saddle, a spiral sink, a spiral source, or a center.

Solve non-homogeneous equation $x' = Ax + b$: See Section 3.3 about reducing $x' = Ax + b$ to $x' = Ax$;

Chapter 4: Second order linear equations

Solve linear homogeneous equations with constant coefficients (use Theorem 4.3.2). Superposition principle (Corollary 4.2.3), linear independence, Wronskian and fundamental set of solutions (Theorem 4.2.6 and Theorem 4.2.7).

Solve non-honogeneous equations using the method of underdetermined coefficients in Section 4.5: (remember Theorem 4.5.1).

Use variation of parameters to solve non-homogeneous linear first order systems of dimension 2 (Theorem 4.7.1); use variation of parameters to solve non-homogeneous linear second order equations (Theorem 4.7.2).

Chapter 5: The Laplace transform

Find the Laplace transform of a given function either using the definition or the table. In exams, if it is not specified whether you need to use the definition or the table, you can use either one. In almost all cases, it is easier to use the table.

Determine whether a function is of exponential order, and find the a.

Find the inverse Laplace transform of a given function.

Solve a differential equation using the Laplace transform.

Section 5.1, 5.2, 5.3, 5.4, discontinuous functions in 5.5 and 5.6 are included.

Chapter 6: Systems of first order linear equations

Theory of first order linear equations - linear independence, principle of superposition, the Wronskian, a fundamental set of solutions

Transform a linear nth order equation into a system of linear equations - see Section 6.2, Page 396

Solve first-order linear homogeneous equations $\mathbf{x}' = A\mathbf{x}$

- when A is nondefective with real eigenvalues - Section 6.3
- when A is nondefective with complex eigenvalues - Section 6.4, use Eq. (4)
- when A is an defective matrix, or A has repeated eigenvalues, use Theorem 6.7.1

Repeated complex eigenvalues are not on the test (Example 4 in Section 6.7).

Compute fundamental matrices.