Midterm1 – 2552, Fall 2018

Instructor: Wenjing Liao

- Test time: 50 minutes.
- Please do not assist another person in the completion of this exam. Please do not copy answers from another student's exam. Please do not have another student take your exam for you. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted.
- No books or calculators are allowed.
- Only one sheet of handwritten note (front and back, letter size) is allowed.
- Read each problem carefully. Show all work for full credit.
- Make sure you have **11 pages**, including the cover page (Page 1), the page of scores (Page 2), and two blank pages (Page 10 and 11).

Your name: _____

Circle your TA's name

Weiwei Zhang Swagath Saraogi Benjamin Ide John Chiles

(1)	
(2)	
(3)	
(4)	
(5)	
Total	

Problem 1 (10 points): Consider the equation

$$\frac{dy}{dt} = y^2(a - y^2).$$

(1) When $a > 0$, find all equilibrium solutions, draw the phase line, and determine whether each critical point is asymptotically stable, semistable, or unstable? (4 points)

(2) When $a = 1$, sketch several solutions on the ty plane (same as integral curves). (3 points)

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(3) When $a < 0$, find all equilibrium solutions, draw the phase line, and determine whether each critical point is asymptotically stable, semistable, or unstable? (3 points)

Problem 2: (10 points): Consider the following equation

$$(ye^{xy}\sin x + e^{xy}\cos x + 2x)dx + xe^{xy}\sin x dy = 0.$$

(1) Show this equation is exact. (4 points)

(2) Find the general solution. No need to write the solution in an explicit form. (6 points)

Problem 3 (10 points): Solve the initial value problem

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix},$$
$$x(0) = 1,\ y(0) = 1.$$

Problem 4 (10 points): Consider the following inhomogeneous system

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -7 \\ 1 \end{bmatrix}.$$

(1) What is the equilibrium solution? (2 points)

(2) Find the general solution of the homogeneous system

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$

(5 points)

(3) Write down the general solution of the inhomogeneous system

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -7 \\ 1 \end{bmatrix}.$$

(1 points)

(4) Draw a phase portrait for the inhomogeneous system. (2 points)

Problem 5 (10 points): Consider the equation

$$\frac{dy}{dt} = y^{\frac{1}{2}}, \qquad y(0) = 0.$$

(1) Find all solutions of this equation. (6 points)

(2) Is there a solution that passes through the point $(2, \frac{1}{4})$? If so, find all solutions that pass through this point. (4 points)